



# VU Research Portal

## the Simple Centern Projection of SU (2) Gauge Theory

Bakker, B.L.G.; Veselov, A.I.; Zubkov, M.A.

### ***published in***

Physics Letters B  
2001

### ***DOI (link to publisher)***

[10.1016/S0370-2693\(00\)01343-5](https://doi.org/10.1016/S0370-2693(00)01343-5)

### ***document version***

Publisher's PDF, also known as Version of record

[Link to publication in VU Research Portal](#)

### ***citation for published version (APA)***

Bakker, B. L. G., Veselov, A. I., & Zubkov, M. A. (2001). the Simple Centern Projection of SU (2) Gauge Theory. *Physics Letters B*, 497, 159-164. [https://doi.org/10.1016/S0370-2693\(00\)01343-5](https://doi.org/10.1016/S0370-2693(00)01343-5)

### **General rights**

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal ?

### **Take down policy**

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

### **E-mail address:**

[vuresearchportal.ub@vu.nl](mailto:vuresearchportal.ub@vu.nl)



ELSEVIER

4 January 2001

Physics Letters B 497 (2001) 159–164

PHYSICS LETTERS B

www.elsevier.nl/locate/npe

# The simple center projection of $SU(2)$ gauge theory

B.L.G. Bakker<sup>a</sup>, A.I. Veselov<sup>b</sup>, M.A. Zubkov<sup>b,\*</sup><sup>a</sup> *Department of Physics and Astronomy, Vrije Universiteit, Amsterdam, The Netherlands*<sup>b</sup> *ITEP, B.Chermushkinskaya 25, Moscow, 117259, Russia*

Received 18 July 2000; received in revised form 31 October 2000; accepted 21 November 2000

Editor: P.V. Landshoff

## Abstract

We consider the  $SU(2)$  lattice gauge model. We propose a new gauge invariant definition of center projection, which we call the Simple Center Projection. We demonstrate the center dominance, i.e., the coincidence of the projected potential with the full potential up to the mass renormalization term at low energies. We also consider the center vortices and the center monopoles (*nexuses*). It turns out that the behavior of such objects qualitatively coincides with the behavior of the vortices and monopoles in the Maximal Center gauge. The connection of the condensation of nexuses with the dual superconductor theory is discussed. Numerically the procedure of extracting the center vortices proposed in this Letter is much simpler than the usual Maximal Center Projection. © 2001 Published by Elsevier Science B.V.

## 1. Introduction

We may need to go a long way before we understand how the confinement mechanism works within non-Abelian gauge theory. A lot of physicists now consider the Abelian projection as the best way to understand the confinement mechanism.

After Abelian projection one should consider the element of some Abelian subgroup of the gauge group instead of the full gauge group element. Thus, the theory becomes Abelian and one can look at the picture of the confinement phenomenon in a simpler way.

Abelian projections differ from each other by the choice of the subgroup of the gauge group and the projection method. The closeness of a given Abelian projection to the solution of the confinement problem

is measured as follows. Suppose that the link gauge group elements  $g_{\text{link}} \in G$  are projected onto the elements of the given Abelian subgroup  $e_{\text{link}} \in E \subset G$ . We consider

$$Z_C = \text{Tr} \prod_{\text{link} \in C} e_{\text{link}}, \quad (1)$$

instead of the Wilson loop and extract the potential from  $Z_C$  (the *projected* potential). If that potential is close to the original confining potential at sufficiently large distances one can say that the projection is suitable for the investigation of the confinement mechanism.

Dealing with the  $SU(2)$  gauge group, the Cartan subgroup  $U(1)$  and the center subgroup  $Z_2$  have been considered. The most popular projections are the Maximal Abelian and the Maximal Center projections. Those projections are achieved by minimization with respect to the gauge transformations of the distance between the given configuration of the link gauge field and the Cartan (center) subgroup of  $SU(2)$ . In

\* Corresponding author.

E-mail address: zubkov@vxitep.itep.ru (M.A. Zubkov).

both cases the potentials are very close to the  $SU(2)$  confining potential but unfortunately do not exactly coincide with it. Greensite et al. [1–3] have argued that only central charges  $q = \pm 1$  are confined in non-Abelian gauge theories.

From the work of Bornyakov et al. [4] it is known that gauge fixing procedures suffer from a gauge copies problem. Several gauges have been used so far in practical computations: the direct and indirect [3] center gauges and the Laplacian center gauge [5]. Only the first two suffer from the occurrence of gauge copies, the last one is free from this difficulty. According to Refs. [1,6] this problem disappears for large lattices and when the number of Gribov copies considered is increased. Of course, this means that the computational effort must also be increased.

In this Letter we propose a new center projection which is not connected with partial gauge fixing. Thus, all the objects existing within the projected theory have a gauge invariant nature. We call this procedure the Simple Center Projection (SCP). Based on numerical simulations we make the conjecture that the projected potential coincides with the  $SU(2)$  potential up to the mass renormalization term at sufficiently large distances.

Within the SCP we can construct the center vortices and the center monopoles (also known as *nexuses*, see Refs. [7] and [8]). The properties of the SCP monopoles found in our investigations lead us to believe that those monopoles are the objects which play the role of the Cooper pairs in the dual superconductor.

## 2. Simple center projection

We consider  $SU(2)$  gluodynamics with the Wilson action

$$S(U) = \beta \sum_{\text{plaq}} (1 - 1/2 \text{Tr} U_{\text{plaq}}). \quad (2)$$

The sum runs over all the plaquettes of the lattice. The plaquette action  $U_{\text{plaq}}$  is defined in the standard way.

First we consider the plaquette variable

$$\begin{aligned} z_{\text{plaq}} &= 1, & \text{if } \text{Tr} U_{\text{plaq}} < 0, \\ z_{\text{plaq}} &= 0, & \text{if } \text{Tr} U_{\text{plaq}} > 0. \end{aligned} \quad (3)$$

We can represent  $z$  as the sum of a closed form  $dN$ <sup>1</sup> for  $N \in \{0, 1\}$  and the form  $2m + q$ . Here  $N = N_{\text{link}}$ ,  $q \in \{0, 1\}$ , and  $m \in \mathbb{Z}$ .

$$z = dN + 2m + q. \quad (4)$$

The physical variables depending upon  $z$  could be expressed through

$$\text{sign Tr} U_{\text{plaq}} = \cos(\pi(dN + q)). \quad (5)$$

We shall say that  $N_{\text{link}}$  is the center projected link variable. There are many different ways to make this projection. The Maximal Center Projection (MCP) uses the gauge ambiguity to make all link matrices  $U$  as close as possible to  $e^{i\pi N}$ . Thus the 1-form  $N$  is fixed for every gauge configuration.

There exist several ways to achieve this [3]. One way, the *direct way*, is to minimize the quantity

$$R = \sum_x \sum_{\mu} \text{Tr}[U_{\mu}(x)] \text{Tr}[U_{\mu}^{\dagger}(x)]. \quad (6)$$

In the *indirect way* one minimizes the quantity

$$R' = \sum_x \sum_{\mu} \text{Tr}[U_{\mu}(x) \sigma_3 U_{\mu}^{\dagger}(x) \sigma_3], \quad (7)$$

and extracts from  $U_{\mu}(x)$  the diagonal part  $A_{\mu} = \exp[i\theta_{\mu}(x)\sigma_3]$  thus fixing the gauge. Finally the remnant Abelian symmetry is used to bring  $A_{\mu}$  as close to an element of  $Z_2$  as possible by maximizing

$$R'' = \sum_x \sum_{\mu} \cos^2 \theta_{\mu}(x). \quad (8)$$

It is clear that both procedures are complicated and time consuming, as the number of variables involved in the case of, e.g.,  $SU(2)$  is three. Whatever method is used, the 1-form  $N$  is fixed for every gauge configuration.

Now we choose a simpler and more natural procedure. Imagine the surface  $\Sigma$  formed by the plaquettes dual to the “negative” plaquettes (for which  $z_{\text{plaq}} = 1$ ). This surface has a boundary. We enlarge the surface by adding a surface  $\Sigma_{\text{add}}$  so that:

- (1) the resulting surface  $\Sigma^1 = \Sigma + \Sigma_{\text{add}}$  will be closed;

<sup>1</sup> We use the formulation of differential forms on the lattice, as described for instance in Ref. [9].

- (2) when we eliminate from the surface  $\Sigma_{\text{add}}$  the plaquettes carrying even numbers  $z_{\text{plaq}} = \dots, -4, -2, 2, 4, \dots$ , the area of the remnant surface will be minimal for the given boundary.

So  $\Sigma^1$  can be represented by the closed form  $dN$  on the original lattice for the integer link variable  $N \in \{0, 1\}$ . And  $N$  is the required center projected link variable.

Numerically this procedure looks as follows. For the given variable  $z$  we should choose such a  $Z_2$  variable  $N$  that  $[dN] \bmod 2$  is as close as possible to  $z$ . It means that we minimize the functional

$$Q = \sum_{\text{plaq}} |(z - dN) \bmod 2|, \quad (9)$$

with respect to  $N$ .

The procedure works in the following way. We consider a given link  $L$  and the sum over plaquettes

$$Q_{\text{link}} = \sum_{L \in \text{plaq}} |(z - dN) \bmod 2|. \quad (10)$$

We minimize this sum with respect to the one link value  $N$ . All links are treated in this way and the procedure is iterated until a global minimum is found.

We call this unique and simple procedure the Simple Center Projection. Our projection method also finds local minima, Gribov copies, but as the procedure is much simpler and faster than the methods used until now, we can afford to repeat our calculations to include several Gribov copies.

The physical meaning of the projected variables becomes clear after considering the continuum limit. Naively, the considered surfaces disappear as the field strength on them tends to infinity. Nevertheless this fact should be investigated more carefully. In any case for reasonable sizes of lattices finite volume effects occur and the scaling window ends at some value of  $\beta$ . Thus the direct drop into the continuum limit is impossible and within the scaling window our surfaces  $\Sigma$  carry large but not infinite field strength. The results of the next section give us the reason to believe that these surfaces are those which play the crucial role in the confinement mechanism.

Let us also mention that it is reasonable to consider the analogous construction for which we change the plaquettes into loops of sizes  $2 \times 2, 3 \times 3, \dots$ . These extended projections solve the problem of the positive plaquette model, for which our “ $1 \times 1$ ”  $\Sigma$  is absent.

### 3. Numerical results

In the calculations we report on here, we used as our standard lattice one with dimension  $24^4$  and for investigations at finite temperature one with size  $24^3 \times 4$ . For reasons of comparison we have occasionally used smaller lattices, of sizes  $12^3 \times 4$  and  $16^3 \times 4$ . Some results were checked on the larger lattice  $32^3 \times 4$ . We have mentioned the gauge copy problem. We checked that for lattices with linear dimensions  $L \leq 24$ , which we have used, 15 copies are sufficient and so we used everywhere 15 Gribov copies.

#### 3.1. Center dominance

We consider the following definition of the projected Wilson loop

$$W_C^{\text{SCP}} = \frac{1}{8} Z_C (3\pi/4)^{\mathcal{P}(C)/4}, \quad (11)$$

where  $\mathcal{P}(C)$  is the perimeter of the loop  $C$ .

It may be interesting for the reader that this expression is very similar to the first term in the character expansion from [6,10]. One can get the above expression from that formula substituting the factor  $1/8$  instead of  $1/4$  and the power  $\mathcal{P}(C)/4$  instead of  $\mathcal{P}$ . However, as the authors of Refs. [6,10] derive their expression for lattices without gauge fixing, they can derive their results with local operators, using the characters of the gauge group to obtain dominance of the fundamental representation. In our case, we perform gauge fixing, which results in a nonlocal operator. Consequently, we cannot use the same derivation as Refs. [6,10]. As we have not been able to find a rigorous derivation of Eq. (11), one must consider our results up till now as empirical ones. It should be stressed that these results were checked for Wilson loops of sizes up till  $6 \times 6$ . It might be useful to check whether Eq. (11) continues to give good results if larger Wilson loops are considered and the statistics is improved.

The reader can recognize from Fig. 1 that  $W_C$  and  $W_C^{\text{SCP}}$  exactly coincide with each other for large enough sizes of the loop (we represent here  $-\log W_C / \text{Area}(C)$  and  $-\log W_C^{\text{SCP}} / \text{Area}(C)$  as a function of the loop area for the values  $\beta = 2.3$  and  $2.4$ ).  $W_C^{\text{SCP}}$  differs from  $Z_C$  by the perimeter factor. Thus we understand that the projected potential (extracted from  $Z_C$ ) differs from the full potential (ex-

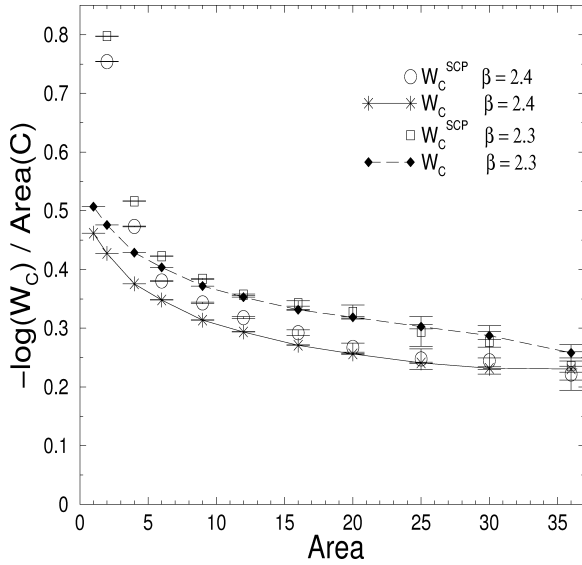


Fig. 1.  $-\log(W_C)/\text{Area}(C)$  for all Wilson loops,  $W_C$ , and computed from projected ones,  $W_C^{\text{SCP}}$ .  $\beta = 2.3$  and  $2.4$ .

tracted from  $W_C$ ) only by the mass renormalization term at low energies. Other terms (including the confining linear term) are the same. It demonstrates the Center Dominance.

### 3.2. The center vortices

We construct the closed two-dimensional center vortices as in [7]

$$\sigma = *dN. \quad (12)$$

First, we can express  $Z_C$  as

$$Z_C = \exp(i\pi \mathbb{L}(\sigma, C)) \quad (13)$$

where  $\mathbb{L}$  is the linking number [7,11].

Thus, according to the previous subsection the Aharonov–Bohm interaction between the center vortices and the charged particle leads to the confinement of the fundamental charge [7,11].

Also we investigate other properties of the center vortices for the finite temperature theory (the nonsymmetric lattice  $24^3 \times 4$ ).

The density of the vortices  $\rho$  is shown in Fig. 2. The fractal dimension, defined as  $D = 1 + 2A/L$ , where  $A$  is the number of plaquettes and  $L$  is the number of links [12] of the vortices, is shown in Fig. 3. A line  $L$

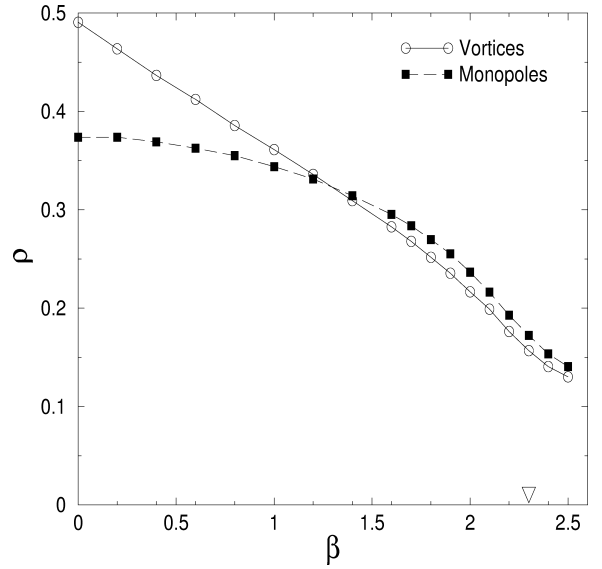


Fig. 2. Density of center vortices and monopoles. The triangle points at the position of the phase transition.

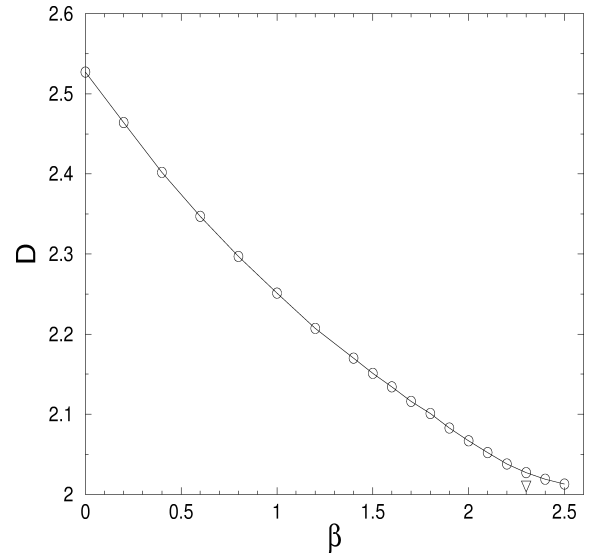


Fig. 3. Fractal dimension  $D$  of the center vortices. The triangle points at the position of the phase transition.

is counted as belonging to the vortex if at least one of the faces of a cube dual to that link contains a plaquette with charge 1.

### 3.3. The center monopoles (nexuses)

Following [7] we construct the center monopoles (nexuses)

$$j = \frac{1}{2} * d[dN] \bmod 2. \quad (14)$$

We show the density of the nexuses as a function of  $\beta$  in Fig. 2. It is important to make sure that the monopole lines are closed. It follows from the equation  $\delta j = \frac{1}{2} * d^{**} d[dN] \bmod 2 = 0$ .

We investigated the percolation properties of  $j$  and considered the probability of two points to be connected by a monopole worldline on a constant-time hypersurface. The dependence of that probability upon  $\beta$  for a nonsymmetric lattice is shown in Fig. 4. The percolation probability is dependent on the lattice size. This is obvious for very small lattices, but it is seen to persist at larger sizes.

Here we would like to remark on the role the monopole condensate plays as an order parameter. Ivanenko et al. [13] have shown for Abelian monopoles, obtained after Maximal Abelian projection, that the monopole condensate can be used as an order parameter: it vanishes in the deconfined phase, while it takes a finite value in the confining phase. The authors of Ref. [13] made the interesting observation that the behavior of the condensate near the point of the phase transition depends on the “thickness” of the monopole lines. It was demonstrated in Ref. [14] that Abelian monopoles are strongly connected with vortices. For vortex lines with a thickness of one lattice spacing, the condensate vanishes smoothly near the critical point, whereas for lines with a thickness of two lattice units, the variation of the condensate near the critical point is very steep.

We see that the monopoles are condensed in the confinement phase and not condensed in the deconfinement phase, but the phase transition is rather smooth as is the case for thin Abelian monopoles. If we would investigate, along the lines of Ref. [13] nexuses of larger sizes like  $2^3$ ,  $3^3$  and so forth, we think that we shall obtain a better sensitivity of the condensates  $C^{(2)}$ ,  $C^{(3)}$  etc. to the phase transition. (Here we use the obvious notation  $C^{(n)}$  for the condensate of vortices of size  $n^3$ .)

We expect the center monopoles to be the monopoles that are present in the dual superconductor picture

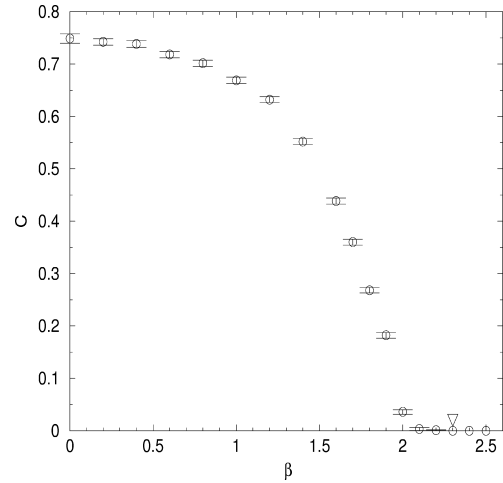


Fig. 4. Probability of connecting two points by a monopole worldline for lattices  $24^3 \times 4$ . The triangle points at the position of the phase transition.

of confinement. As the phase transition is not very pronounced for the thin vortices we considered here, our results may be taken as a “proof of principle”. They must be substantiated by considering thick vortices which are supposed to be more strongly connected to confinement [15].

The analytical connection of the monopole condensation and the formation of the dual superconductor was considered in [16] for the case of  $SU(3)$  symmetry. Of course, the results of that paper obtain also for the  $SU(2)$  theory. It follows from [16] that in the case both condensation of nexuses and center dominance occur, the picture of the dual superconductor in which the nexuses play the role of Cooper pairs and the quarks play the role of the monopoles becomes clear.

In particular, one can rewrite the fundamental Wilson loop as follows

$$\begin{aligned} \langle W(C) \rangle &= \int_{-\pi}^{\pi} DH \int D\Phi \in C \Phi \\ &\times \exp \left( -Q(dH + \pi^* A[C]) \right. \\ &\quad \left. - \sum_{xy} \Phi_x e^{2iH_{xy}} \Phi_y^+ - V(|\Phi|) \right). \quad (15) \end{aligned}$$

Here  $H$  is the electromagnetic field,  $A[C]$  is the area of the surface spanned on the quark loop,  $\Phi$  is the nexus field,  $Q$  is nonlocal effective action,  $V$  is an infinitely deep potential, supporting the infinite value of the nexus condensate.

Thus we have indeed the nonlocal relativistic superconductor theory, in which the nexuses are the Cooper pairs and the quarks are the monopoles. The condensation of nexuses gives rise to the formation of the quark–antiquark string appearing as the Abrikosov vortex.

#### 4. Conclusions

In this Letter we construct the gauge invariant Center projection and show that center dominance takes place. We investigate the properties of the topological defects in the center projected theory, which are shown to be closely connected to the confinement picture. Particularly it occurs that the center monopoles from this projection are good candidates for the Cooper pairs in the dual superconductor. The simple numerical nature of the Simple Center Projection, the exact Center Dominance and the properties of the topological defects existing in the center projected theory give us the reason to propose SCP as the basic Abelian projection for the considering of the confinement picture.

#### Acknowledgements

We are grateful to V.G. Bornyakov, M.N. Chernodub, and M.I. Polikarpov for useful discussions. A.I.V. and M.A.Z. kindly acknowledge the hospitality

of the Department of Physics and Astronomy of the Vrije Universiteit, where part of this work was done. This work was partly supported by the grants RFBR 99-01-01230, INTAS 96-370.

#### References

- [1] J. Greensite, Talk at Confinement 2000, Osaka, Japan, March 7–10, 2000, hep-lat/0005001.
- [2] L. Del Debbio, M. Faber, J. Giedt, J. Greensite, S. Olejnik, Phys. Rev. D 58 (1998) 094501.
- [3] L. Del Debbio, M. Faber, J. Greensite, S. Olejnik, hep-lat/9802003.
- [4] V.G. Bornyakov, D.A. Komarov, M.I. Polikarpov, A.I. Veselov, Talk at Confinement 2000, Osaka, Japan, March 7–10, 2000, hep-lat/0002017.
- [5] Ph. de Forcrand, M. D’Elia, Phys. Rev. Lett. 82 (1999) 4582; C. Alexandrou, M.M. D’Elia, Ph. de Forcrand, hep-lat/9907028.
- [6] M. Faber, J. Greensite, S. Olejnik, D. Yamada, hep-lat/9912002.
- [7] M.N. Chernodub, M.I. Polikarpov, A.I. Veselov, M.A. Zubkov, Nucl. Phys. Proc. Suppl. 73 (1999) 575, hep-lat/9809158.
- [8] G.E. Volovik, Pisma Zh. Eksp. Teor. Fiz. 70 (1999) 776, cond-mat/9911374.
- [9] M.I. Polikarpov, U.J. Wiese, M.A. Zubkov, Phys. Lett. B 309 (1993) 133.
- [10] M.C. Ogilvie, Nucl. Phys. Proc. Suppl. 73 (1999) 542, hep-lat/9809167.
- [11] M.N. Chernodub, M.I. Polikarpov, A.I. Veselov, Phys. Lett. B 399 (1997) 267, hep-lat/9610007.
- [12] B.L.G. Bakker, A.I. Veselov, M.A. Zubkov, Phys. Lett. B 471 (1999) 214, hep-lat/9902010.
- [13] T.L. Ivanenko, A.V. Pochinsky, M.I. Polikarpov, Phys. Lett. B 302 (1993) 458.
- [14] J. Ambjørn, J. Geidt, J. Greensite, hep-lat/9907021.
- [15] M. Faber, J. Greensite, S. Olejnik, hep-lat/0005017.
- [16] M.A. Zubkov, hep-lat/0001034.